<u>Scientific Measurements:</u> Significant Figures and Statistical Analysis

Purpose
The purpose of this activity is to get familiar with the approximate precision of the equipment in your laboratory. Specifically, you will be expected to learn how to correctly

- record measurements with an appropriate number of significant figures,
- manipulate measured values when performing basic mathematical operations (+/−/×/÷),
- distinguish between accuracy and precision, and
- report and interpret an average and standard deviation for a set of data.

Background
If you measure your weight at home on your bathroom scale, you may get a reading of 135 lb. At a doctor's office 30 minutes later, the nurse measures your weight to be 145 lb. The scale at the doctor's office is probably more ***accurate*** than the one you have at home. Perhaps it was calibrated, whereas yours wasn't.

On the other hand, suppose you are at home and your new digital scale reads 134.8 lb. You step off, and step back on. It reads 134.6 lb. Stepping on and off the balance a few times leads to slightly different values each time. As long as the values tend to be around a certain value, being slightly off is not "wrong". However, the closer the values are to each other, the higher ***precision*** has been achieved.

As a scientist, gaining an understanding of the accuracy and precision of measurements is important. Here are some highlights:

- Every measurement contains some degree of error. No measurement is ever exact.
- In cases where a "true value" is not provided, or you are to determine a value experimentally, the mean (average) will be your best value for a measurement.
- The percent error is used to compare an experimental value with a "true" value. The smaller the percent error, the greater the accuracy.
- The standard deviation (stdev or SD) and the relative standard deviation (rel. stdev or RSD) are used to discuss precision. The smaller the standard deviation, the higher the precision.

Accuracy vs. Precision
Precision and accuracy are terms that are often misused – they are not interchangeable and have very different meanings in a scientific context.

Accuracy is a measure of the correctness of a measurement. For example, the density of zinc at 25 °C is 7.14g/mL. Experimentally, you might determine the density of a piece of zinc to be 7.27 g/mL, but another student in the class may calculate the density of zinc to be 6.56 g/mL (assuming 25 °C). Since your answer is closer to the agreed upon value, your measurement is more accurate.

How is accuracy measured? We express how accurate our results are as a % error. You can compare the extent of error in your experimental readings by using the following formula:

$$\%\ \text{error} = \left|\frac{\text{experimental - true}}{\text{true}}\right| \times 100\%$$

If you use the values given above, you will find that your zinc density gives you a % error of |(7.27-7.14)/7.14| x 100 = 1.82%, while your classmate got a % error of |(6.56-7.14)/7.14|x100 = 8.12%. This means you were more accurate. **For many experiments, an error of 10% is an acceptable range for accuracy of your results**, and a higher error might indicate that you might have problems with your technique, reagents, or equipment (though the acceptable % error varies with the type of experiment you may do – some experiments are very sensitive to error and may result in a % error that is reasonably higher than others).

Often there is no "true" value to compare to in an experiment as we had above and therefore you cannot comment on a measurement's accuracy. In these cases you will do several trials or multiple experiments and the average (or mean) will be taken to be your "true" value.

To calculate a mean, you sum all the values of your trials and divide by the number of trials... something you have probably done many times. Let's translate this into statistics lingo, where *x bar* is the mean, *n* = number of trials, capital sigma (Σ) means "sum", and *x* is the value obtained for each trial, from 1 through *n*.

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{n}(x_1 + \cdots + x_n)$$

Precision is a measure of the reproducibility of a measurement. Imagine you repeated the zinc density experiment a second time and this time measured a value of 7.26 g/mL, and a third time it was 7.29 g/mL. Your value is very close to your first experiment. You could say that your precision is quite good but how do you quantify it?

How is precision measured? Conceptually, you can see that the less the values deviate (or differ) from each other, the higher the precision. We will use a statistical measure called a standard deviation.

To calculate a standard deviation (σ), you take each trial value, x_i, subtract it from the mean, *x bar*, and square it to get a variance. Then you add all of these variances for every trial to get a sum of variances. Then you divide by *N*-1 where *N* = number of trials. Then you take a square root. In statistical lingo, it looks like this:

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \overline{x})^2}.$$

With more than just a few trials, this calculation can be tedious and is much easier to do with your calculator or Excel – ask your instructor or refer to your calculator manual for help (or a Google search). If you calculate a standard deviation for your density values 7.27 g/mL, 7.26 g/mL, and 7.29 g/mL, you will need the average (7.27 g/mL). The standard deviation is:

$$\boldsymbol{\sigma} = \sqrt{\left[\frac{(7.27-7.27)^2+(7.26-7.27)^2+(7.29-7.27)^2}{3-1}\right]} = \mathbf{0.02\ g/mL}$$

How do I report my experimental results? For your density experiment, you would report both the mean and the standard deviation in this format: *x bar* ± σ. Therefore, you would report 7.27 ± 0.02 g/mL as your final result. Comparing another student's results, 7.18 ± 0.15 g/mL, you could say their result was more accurate (7.18 g/mL is closer to 7.14 g/mL, the true value of the density of zinc) but their precision (± 0.15 g/mL) was not as good as yours (± 0.02 g/mL).

What does the standard deviation mean? There is a lot to understand about statistics to answer this question that fall outside the scope of this course. For simplicity, we can say that your measurement of 7.27 ± 0.02 g/mL means that individual measurements of density will likely be within +0.02 g/mL or -0.02 g/mL of the mean. That means your collected measurements should fall within the range 7.25 g/mL – 7.29 g/mL most of the time. Since the range is rather small, we say the precision/reproducibility is good.

What is a "good" standard deviation? How small is "small", and how large is "large"? To answer this question, you can calculate a relative standard deviation, which like % error, gives you a value relative to the mean and is expressed in %.

$$\textbf{Relative standard deviation (RSD)} = \left(\frac{\sigma}{mean}\right) \textbf{ x 100\%}$$

In our zinc density example, the standard deviation was 0.02 g/mL and the mean was 7.27 g/mL. This means the RSD is (0.02 / 7.27) x 100 = 0.3%. For our purposes, we will consider an RSD of 10% to be rather small. The result has high precision (<10% RSD).

> ***NOTE: The 10% guideline for % error and RSD are just guidelines. Please do not start an experiment over if your results do not follow the guidelines. Always ask your instructor before discarding results. Do not start an experiment over without permission from your instructor due to time constraints.***

So far we've taken you through a lot of statistics and calculations. In real life, you don't normally measure something three times and always take an average and standard deviation. That would be very tedious! For example, when you weigh yourself, you rarely get on the balance three times and do the calculations. Something about the balance can tell you how good the measurement is, and the level of precision in the measurements obtained with only one trial.

In addition to reproducibility, precision also deals with the closeness or fineness with which a measurement may be made. What does that mean? Think about measuring your weight. If you measure your weight at home on your bathroom scale, you may get a reading of 135.5 lb. At a doctor's office that day the nurse measures your weight to be 135.39 lb. The 135.39 value has digits out to the hundredths place whereas the 135.5 value only has digits out to the tenths place; therefore, the scale at the doctor's office has a higher precision than the one you have at home.

Engineers and some scientists represent the precision of a piece of equipment by writing a "± error" after the measurement. If trials were done, you might calculate the standard deviation and use that value for the ± error. But if you are taking one measurement, you might estimate what that ± error value is based on the graduations of the measuring tool. For example, the first scale's measurement could be written as 135.5 ± 0.1 lb. This shows that there is an uncertainty in the last digit of the measurement. The doctor's scale measurement would be written as 135.39 ± 0.01 lb. (For our purposes, we will assume the unit of "1" for the decimal place of the last written digit.)

Chemists use significant figures (sig figs) to indicate the precision without having to record the ± error of the equipment used to make the measurements**.** There is an implied understanding that the last recorded digit contains some level of error. Instead of writing the ± error explicitly, we use significant figures to communicate the precision of the measuring device.

Significant figures include all the known values of a measurement plus one guess. Let's go back to the scale example. When you stand on the bathroom scale, the needle might point between 135 lb and 136 lb (as in figure 1 below). You know that you weigh more than 135 lb but less than 136 lb. The correct way to report this value is to report the known values (135) plus a guess (135.*4* ← the 4 is the guess).

In general, estimate the value to **one decimal place** more than the level of graduation.
(Or, in other words, 1/10 of the smallest division you can see on the scale!)

In the example above, the graduation is every 1 lb. Therefore, the measurement is reported to the 0.1 lb (135.4 lb, which has one decimal place).

Every measurement you take must include the proper number of significant figures!! You can determine this by looking at the graduations on your device. The last value is always a guess made by you. Don't hesitate, just guess. This makes the last digit the uncertain digit. Many students ask if significant figures are important.

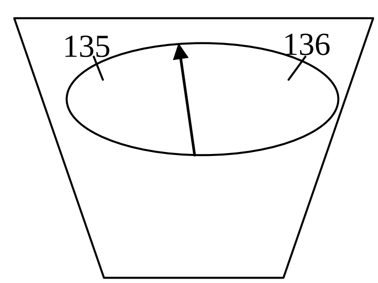

Figure 1 – Scale showing a mass between 135 and 136 lb. Note the lack of markings between 135 and 136.

Read the following fable and then draw your own conclusions:

Are Significant Figures Important?

A student once needed a cube of metal which had to have a mass of 83 grams. He knew the density of this metal was 8.67 g/mL, which told him the cube's volume. Believing significant figures were invented just to make life difficult for chemistry students and had no practical use in the real world, he calculated the volume of the cube as 9.573 mL. He thus determined that the edge of the cube had to be 2.097 cm. He took his plans to the machine shop where his friend had the same type of work done the previous year. The shop foreman said, "Yes, we can make this according to your specifications - but it will be expensive."

"That's OK," replied the student. "It's important." He knew his friend has paid $35, and he had been given $50 out of the school's research budget to get the job done.

He returned the next day, expecting the job to be done. "Sorry," said the foreman. "We're still working on it. Try next week." Finally the day came, and our friend got his cube. It looked very, very smooth and shiny and beautiful in its velvet case. Seeing it, our hero had a premonition of disaster and became a bit nervous. But he summoned up enough courage to ask for the bill. "$500, and cheap at the price. We had a terrific job getting it right -- had to make three before we got one right."

“But--but--my friend paid only $35 for the same thing!"

"No. He wanted a cube 2.1 cm on an edge, and your specifications called for 2.097. We had yours roughed out to 2.1 that very afternoon, but it was the precision grinding and lapping to get it down to 2.097 which took so long and cost the big money. The first one we made was 2.089 on one edge when we got finshed, so we had to scrap it. The second was closer, but still not what you specified. That's why the three tries."

"Oh!"[1]

So, what do you think? Are sig figs important for communicating information about precision? Accuracy? Both?

[1]http://dbhs.wvusd.k12.ca.us/webdocs/SigFigs/SigFigsFable.html

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Report Sheets Name__________________ Date________
Scientific Measurements Lab partner______________Section______

Instructions: Turn in these sheets only. (Detach the previous pages.)

Grading Criteria - You will earn/lose points based on the following:

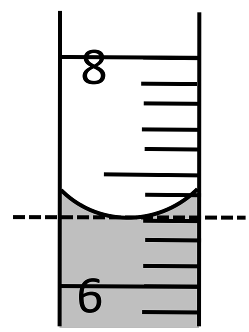
- Number of significant figures used, based on the measurement tool
- Including units for all measured values
- Explanations where required
- Use of a **pen** for recorded data
- Do not white out or erase data, use a strikethrough

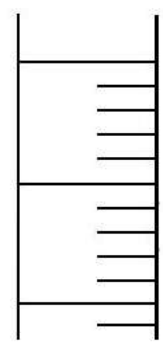
Procedure and Data

Part 1: Recording measurements to the correct number of significant figures.

1) a) Take out a 10-mL graduated cylinder. Check to see whether your graduated cylinder is graduated to every 0.1 mL or every 0.2 mL.

My 10-mL cylinder is graduated to every ______ mL.





Since the graduations are in the tenths place (one decimal place) in either case, you should report your volume to the hudredths place (two decimal places).

For example, acceptable measurements for the following measurement (pictured at right) would be between 6.60 and 6.62 mL.

b) Place about 5 to 6 mL of water into your 10-mL cylinder and make a sketch of it on the blank scale provided above. Include **at least two labeled graduations** and clearly show the curved meniscus (as in the previous drawing). Be as accurate in your drawing as possible.

c) Report your measurement **WITH UNITS** using the correct number of significant figures: ___________

2) a) Find a plastic ruler and a pen or pencil.

The smallest graduation of the plastic ruler represents ________ cm and ______ mm.

b) Use the plastic ruler to measure the length of your pencil/pen. **Note the smallest graduation and make sure you <u>record one uncertain digit</u>.**

My pencil length in centimeters is ______________________ cm.

c) Convert your measurement from centimeters to millimeters. Since you are using the same measuring device, your answer should contain the same number of significant figures. Report your answer to the correct number of significant figures.

The pencil is ________________ centimeters long, or ______________ millimeters.

d) Obtain a meter stick and record the length of the same pencil in meters.

The smallest graduation of the meter stick represents ________ cm and ______ mm.

My pencil length in meters is ______________________ m.

Based on the information so far, does your measurement have a different precision when you use a meter stick as compared to the plastic ruler? State yes or no, and explain.

____

____

3) a) Obtain a penny. Use the balance to measure the mass of the weight. With any digital device, including digital balances, always record every digit on the display. **Record units**.

Mass _________________

b) Now measure several other pennies and note their masses. Based on your observations, to what decimal place does this balance report? (Circle your choice)

10g 1g 0.1 g 0.01 g 0.001g 0.0001g

4) If the mass of an object was measured on the balance from question 3 and a classmate said that the reading was 120.1 g, would this be correct? **No, it would be incorrect because the balance measures more sig figs than that!** Unlike in math class where trailing zeros are often neglected, zeros may need to be included at the end of a number to indicate the precision of the device.

a) The correctly reported mass should be = __________ g instead of 120.1 g

b) What is the implied error range for the following measurements:
120.10 g ± _____g vs. 120.1 g ± _____g

c) Is there a difference in reporting 120.1 g versus 120.10 g? Yes / No

d) Which measurement above implies a greater level of precision? **Explain** briefly.

****** If you have any questions, ask your instructor before you proceed.******

Part 2: Significant Figures in Calculations

1) Take a 150-mL Erlenmeyer flask and fill it to the 150-mL mark with tap water.
2) Obtain four differently sized graduated cylinders: 10-, 25-, 50-, and 100-mL graduated cylinders. Pour between 8 to 9 mL of the water into the 10-mL cylinder, then 23 to 24 mL of the water into the 25-mL cylinder, then 48 to 49 mL of the water into the 50-mL cylinder, and then the remaining water into the 100-mL cylinder. Record the volume in each cylinder, reporting each value to the appropriate number of significant figures.

Graduated Cylinder	Volume (mL)
10-mL	
25-mL	
50-mL	
100-mL	

3) When performing addition (or subtraction) calculations using measured values, the calculated value is reported to the same decimal place as the least precise measurement. Calculate the sum of the measured volumes to obtain the total volume and *report this to the correct number of significant figures. Include units!*

    Volume of water = ____________

4) Did your Erlenmeyer flask really hold precisely 150 mL when filled to the 150-mL mark?

5) Using the volume of water calculated above as your "true volume", calculate a percent error for the 150-mL Erlenmeyer flask. Show your work below.

6) Based on your answer, would you say an Erlenmeyer flask is better suited for measuring volumes (<5% error) or for holding and mixing solutions?

7) When performing multiplication or division calculations using measured values, the calculated value is reported with the same number of significant figures as the least precise measurement. If any of the values used in the calculation are actually the results of previous calculations, such as the total volume calculated above, use an unrounded value, or at least a value with two extra digits, in the calculation to prevent rounding errors. Only round at the last step.

For the following questions, report your answers with the correct number of significant figures.

(a) If the density of water is 0.9982 g/mL at 20°C, what is the mass of water that was in the Erlenmeyer flask? ___________

(b) If the same flask is filled with pure ethanol whose density is 0.78 g/mL, what will be the mass of ethanol? ______________

Part 3: Accuracy vs. Precision

1) Take out a 100-mL beaker and make sure it is clean and dry. Record the mass of the beaker empty: ___________ g

   a) Examine the graduations. The 100-mL beaker is graduated every _____ mL.

   b) Since we estimate one decimal place more than the level of graduation (or 1/10 of the smallest graduation), the measurement should be recorded to every ______ mL.

   c) Add about 30-40 mL of water to your beaker, using the graduations on the beaker.

   d) Measure the volume of this water using the beaker and record it below.

_______________ **mL (using a 100-mL beaker)**

   e) Which digit contains the uncertain value? Circle the digit above that was estimated.

   f) How many significant figures can you report based on the graduations? _______ sig figs

2)
a) Measure the mass of an empty 100-mL graduated cylinder: _______________ g
Make sure you use the correct number of significant figures!

b) Now take the contents of your beaker (the 30-40 mL of water) and pour it into the 100-mL graduated cylinder. Thinking about what you learned earlier (looking at the graduations), record a measurement for the volume of water.

_______________ mL(using a 100-mL graduated cylinder)

c) Again, circle the digit that was estimated.

d) How many sig figs did you report? ______ sig figs

3) Examine the beaker and the graduated cylinder. Look at your recorded measurements (in the boxes above). Which glassware will give a more ***precise*** measurement? **Explain.**

4)

a) Record the mass of the 100-mL cylinder plus the 30-40 mL of water: _______________ g

b) Calculate the mass of the 30-40 mL of water alone: ______________ g

c) Calculate the **TRUE volume** of 30-40 mL of water using the density of water (0.9982 g/mL at 20.0°C), the mass of water calculated above, and this equation: $density = \frac{mass\ (g)}{volume\ (mL)}$

Show your calculation and use the proper number of significant figures:

5) Which piece of glassware, the beaker or the graduated cylinder, do you expect to be more ***accurate***? **Explain.**

6)

a) Calculate the percent error for volumes measured from each glassware using the % error equation in the background section of this lab. Use the values in boxes above for your "experimental" values, and the **true volume** above. SHOW YOUR WORK.

% $error_{beaker}$ = _____________% % $error_{graduated\ cylinder}$ = ____________%

b) Based on the calculated % error values, which is more ***accurate***, the volume measurement using the beaker or the graduated cylinder? Does this agree with your prediction in the question above? (It is expected these answers will agree for most students, however, if they do not, give a reason why not or go back and check your results/calculations.)

7) Given a choice, which glassware would you use to measure volumes more ***precisely***, a graduated cylinder or a beaker? Explain briefly.

8) Many times you'll come across a new piece of lab equipment and need to determine its precision. Without performing any calculations, what would you look at to determine the precision of a piece of glassware?

Part 4: Average and Standard Deviation

1) Take a 50 mL beaker and record its mass.

Mass of empty beaker: __________________g

2) Measure 1.00 mL of distilled water using a 5-mL graduated pipette and a pipette pump and transfer it to the weighted 50-mL beaker. Record the mass of beaker and water. (The green pipette pumps are used for 5- and 10-mL pipettes. The blue pumps are used for 1-mL pipettes.)
3) Repeat step 2 adding 1.00 mL water each time and recording the mass of beaker and water until you have added a total of 5.00 mL of water. Record the data in the following table.
4) Calculate the mass of water for each volume. Use the volume and calculated mass to calculate the density of water. Report values to the *correct number of significant digits*.

Volume of water (mL)	Mass of beaker + water (g)	Mass of water (g)	Density of water (g/mL)
1.00			
2.00			
3.00			
4.00			
5.00			

5) Calculate the average, standard deviation, and relative standard deviation of your results. Show your work and report your answers.

6) Based on your calculated average density and given that true density of water is 0.9982 g/mL (at 20°C), calculate the percent error and comment on the ***accuracy*** of your data.

7) Use the standard deviation and relative standard deviation to comment on the ***precision*** of your data.

Follow up Questions

Always report your answers using significant figures!

1) a) Suppose a 10-mL pipet has an uncertainty of about ± 0.01 mL. If it is filled with to the mark (10-mL), the volume should be reported as (include the necessary decimal places):

_______________ mL

b) A liquid is filled to the 100-mL mark of a 100-mL graduated cylinder. How should the volume be recorded? The graduations on a cylinder are every 0.1 mL.

_______________ mL

2) If your 100-mL graduated cylinder (the same one used above) is filled to 35.0 mL with pure ethanol whose density is 0.79 g/mL at 20.0°C, calculate the mass of ethanol. Show your work.

_______________ g

3) What is the sum of the volumes 35.0 mL, 21.25 mL, 151 mL?

_______________ mL

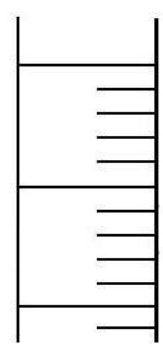
4) Fill in the blanks with the letters corresponding to the correct terms. Each term is used only once.

(a) true value	(c) reproducibility	(e) relative standard deviation (rsd)
(b) accuracy	(d) precision	(f) standard deviation (stdev)

Percent error is one way to evaluate the ___________ of a measurement, and indicates how close a measurement is to its ___________. The ___________ is calculated to discuss precision, and describes the _________of a measurement. Standard deviation indicates the spread of data, and when divided by the mean and expressed in %, it is called the ________. When this value is less than 10%, it indicates a high level of _______________ in the data.

5) Mark the glassware's scale so that it can ***report to*** the thousandths place. "Report to" means to record digits to this decimal place, including your estimated digit.

HINT: A common *mistake* is to draw devices graduated to the thousandths place—don't do this!



Name_________________________ Section_____

Pre-Lab Assignment—To be completed BEFORE lab!
Scientific Measurements

Read the background section of this experiment. Answer the following questions.

1) A student performed an analysis of a sample for its calcium content and obtained the following results:

14.92 g, 14.91 g, 14.88 g, 14.92 g

The actual amount of calcium in the sample is 20.90 g. For this lab, the instructor asked students to obtain results within 5% error and RSD and to redo the experiment if that was not achieved.

(a) Report the student's results including the mean and the standard deviation, considering all the data points collected. Show your calculations below. Do the calculations by hand, and if you have a calculator or Excel you can use it to check your answer.

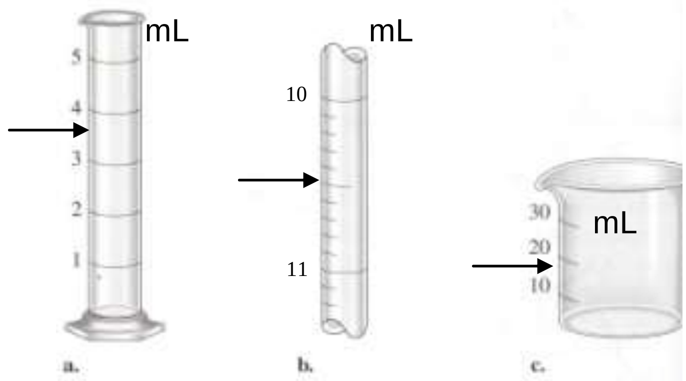
_________g ± ______g
(average) (standard deviation)

(b) Calculate the percent error for this student's experiment.

(c) Calculate the relative standard deviation for this student's experiment.

(d) Briefly comment on the accuracy vs. the precision obtained by the student. Does the student need to redo the experiment?

2) (a) Provide a measurement for each of the following pieces of glassware filled to the level indicated by the arrow. Use the correct number of significant figures in your answer. Always use units. **Remember to estimate one digit more than the level of graduation.**



(See note below*)

*Buret markings are opposite to that of cylinders and beakers.

(b) Which gives the most precision, (a.) the graduated cylinder, (b.) the buret, or (c.) the beaker?

(c) How can you tell by simply looking at the glassware? **Explain briefly**.

3) (a) Label this beaker with values so that it can ***report*** volume to the **tens (not tenths)** place. ("Report to" means to record digits to this decimal place, including your estimated digit.)

(b) Give a sample measurement that could be obtained from this beaker.
Use mL for units. Use significant figures.

4) In your own words, what is the moral of the story told in the Background section?